7.5 - Solving Rational Equations

Warmup

1. $\dfrac{x^2-y^2}{y^2}\cdot\dfrac{y^3}{y-x}$

$-y(x+y)$

2. $\dfrac{x^2-y^2}{x+y}\cdot\dfrac{11}{x-y}$

11

3. $-\dfrac{x^2-y^2}{x+y}\cdot\dfrac{1}{x-y}$

-1

4. $\dfrac{(y-2)^2}{(x-4)^2}\cdot\dfrac{x-4}{y-2}$

$\dfrac{y-2}{x-4}$

5. $\dfrac{a^2-b^2}{14}\cdot\dfrac{35}{a+b}$

$\dfrac{5(a-b)}{2}$

6. $\dfrac{x^2-y^2}{70}\cdot\dfrac{56}{4x-4y}$

$\dfrac{x+y}{5}$

7.5 - Solving Rational Equations

$$\frac{x}{3}+\frac{5}{6}=\frac{3}{2}$$

$$6\left(\frac{2x}{6}+\frac{5}{6}=\frac{9}{6}\right)$$

$$2x+5=9$$

$$2x=4$$

$$x=2$$

$$\frac{2x}{6}+\frac{5}{6}=\frac{9}{6}$$

Practice

1. $\frac{x}{3}-\frac{3}{4}=\frac{5}{6}$

$x=\frac{19}{4}$

2. $\frac{x-2}{2}-\frac{x-1}{5}=\frac{1}{4}$

$x=\frac{7}{2}$

3. $\frac{2x-1}{6}=\frac{x+2}{4}+\frac{1}{3}$

$x=12$

7.5 - Solving Rational Equations

$$\frac{3}{x^2-7x+10}+2=\frac{x-4}{x-5}$$

$$\frac{3}{(x-2)(x-5)}+2=\frac{x-4}{x-5}$$

$$\frac{3}{(x-2)(x-5)}+\frac{2\cdot(x-2)(x-5)}{(x-2)(x-5)}=\frac{(x-4)\cdot(x-2)}{(x-5)\cdot(x-2)}$$

$x \neq 2,5$

$$3+2\cdot(x^2-7x+10)=x^2-6x+8$$

$$3+2x^2-14x+20=x^2-6x+8$$

$$x^2-8x+15=0$$

$$(x-3)(x-5)=0$$

$x = 3, \cancel{5}$

7.5 - Solving Rational Equations

Practice

1. $\frac{3}{x+1} - \frac{1}{x-2} = \frac{1}{x^2-x-2}$

$x = 4$

2. $\frac{t}{t-1} = \frac{1}{t+2} + \frac{3}{t^2+t-2}$

no solution

3. $\left(\frac{x-3}{x+1}\right)^2 = 2 \cdot \frac{x-3}{x+1} + 3$

$x = 1, -3$

7.5 - Solving Rational Equations

$$\frac{3x}{4} + 1 > \frac{x-1}{2}$$

$$\frac{3x}{4} + \frac{4}{4} > \frac{2(x-1)}{4}$$

$$3x + 4 > 2x - 2$$

$$x + 4 > -2$$

$$x > -6$$

Practice

1. $\frac{x+1}{6} < x - \frac{3x-2}{4}$

$x > -4$

2. $\frac{(x+1)(x-2)}{(x+3)(x+4)} \geq 0$

$x < -4$ *or* $-3 < x \leq -1$ *or* $x \geq 2$

What about? $\frac{(x-7)(x+1)(x-2)}{(x-7)(x+3)(x+4)} \geq 0$

$x < -4$ *or* $-3 < x \leq -1$ *or* $x \geq 2$ *and* $x \neq 7$

7.5 - Solving Rational Equations

Find two positive integers that differ by 6, and whose reciprocals add to 1/4.

Integer one = x

Integer two = $x - 6$

$$\frac{1}{x} + \frac{1}{x-6} = \frac{1}{4}$$

$$4(x-6) + 4x = x(x-6)$$

$$4x - 24 + 4x = x^2 - 6x$$

$$0 = x^2 - 14x + 24$$

$$0 = (x-12)(x-2)$$

$x = 12, 2$ ✗

Two integers are 6 and 12

7.5 - Solving Rational Equations

Practice

Find two positive integers that differ by 2, and whose reciprocals differ by 1/40.

Remember: $\frac{1}{x-2}$ is bigger than $\frac{1}{x}$!!

Two integers are 8 and 10

7.5 - Solving Rational Equations

Domain and Zeros

$$\frac{x^2-3x-4}{x^2-1}$$ What is the domain? $x \neq 1, -1$

$$\frac{(x-4)(x+1)}{(x-1)(x+1)}$$ What are the zeros? $x = 4, -1$

Practice - Simplify, find domain and zeros

$$\frac{x^2+11x}{x-2} \cdot \frac{1}{3x^2+6x} \cdot \frac{x^2-4}{x+11} = \frac{1}{3}$$

$D: \ x \neq -2, 0, 2, -11$

$Z: \ none$

7.5 - Solving Rational Equations

Work Rate Problem

If Ravi can paint a house in 10 hours and Samantha can paint that same house in 15 hours, how long will it take if both painted the house together?

$t = time\ for\ combined$

	Rate	Time (hrs)	Houses Painted
Ravi	1 house/10 hours	t	t/10
Samantha	1 house/15 hours	t	t/15
Combined	1 house/t hours	t	t/t = 1

$$\frac{t}{10} + \frac{t}{15} = 1$$

$$30\left(\frac{t}{10} + \frac{t}{15} = 1\right)$$

$$3t + 2t = 30$$

$$5t = 30$$

$$t = 6\ hours$$

7.5 - Solving Rational Equations

Work Rate Problem

It takes pump A alone 6 hours to fill a tank. It takes pump B alone 8 hours to fill the same tank. We want to use three pumps, A, B, and C to fill the tank in 2 hours. What should be the rate of pump C? How long would it take for pump C to fill the tank alone?

$$rate = \frac{5}{24} tanks/hr$$

$$\frac{24}{5} = 4.8 \ hours$$